



Martini Workshop 2015 Coarse Graining Basics

Alex de Vries

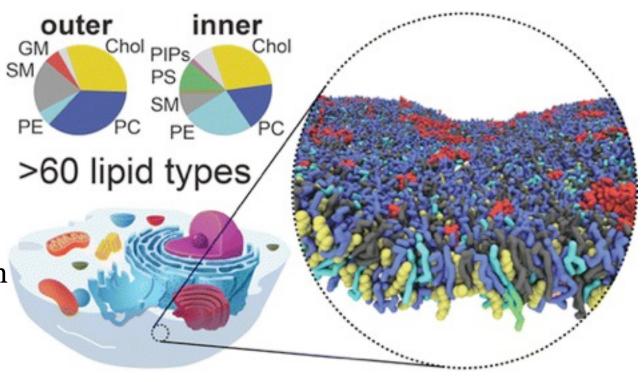
Every word or concept, clear as it may seem to be, has only a limited range of applicability

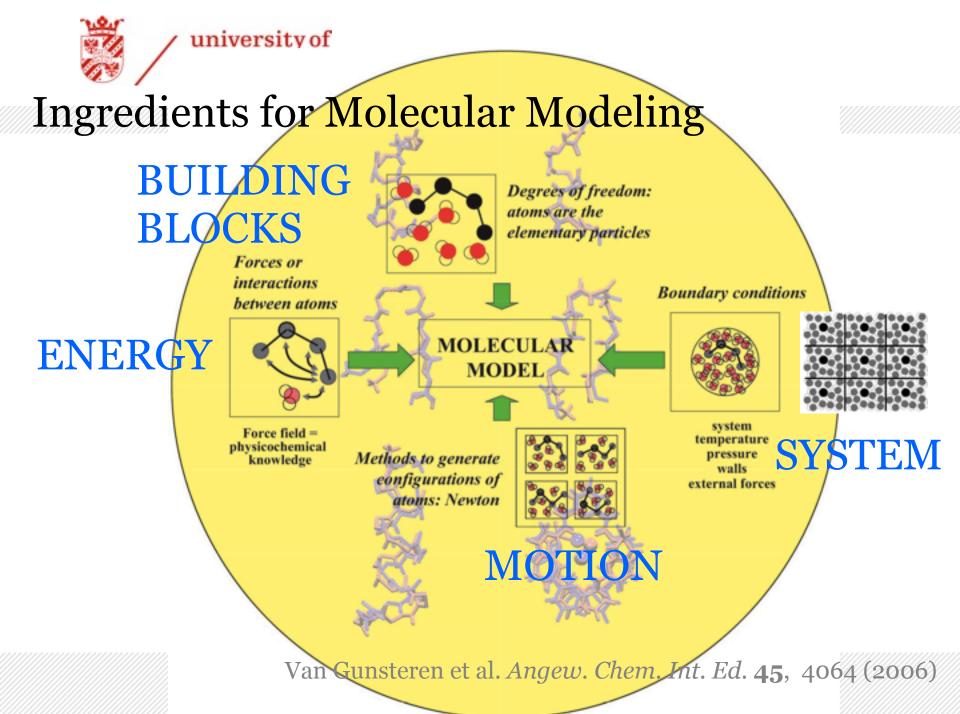
Werner Heisenberg



Recent MARTINI work

- Mammalian membrane simulation of unprecedented complexity and duration.
- 63 different lipids, asymmetric layers.
- ~0.5 Mbeads, 40 μs
- enables the highresolution study of the distribution and dynamics of lipids interacting with each other.
- the next step is to add proteins.

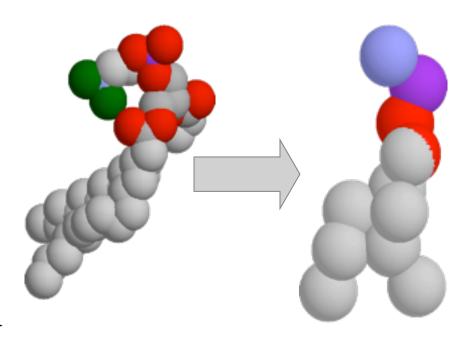






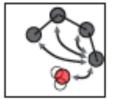
Popular Molecular Models

- All Atoms (OPLS-AA/L, CHARMM, AMBER)
 - Each atom is treated as a particle
- > United Atoms (GROMOS)
 - Each atom is treated as a particle, except aliphatic CH, CH₂, CH₃
- Coarse grained superatoms (MARTINI)
 - A group of (~4 for Martini) united atoms is treated as a particle



 DPPC molecule (a lipid) in UA and CG (MARTINI) representations



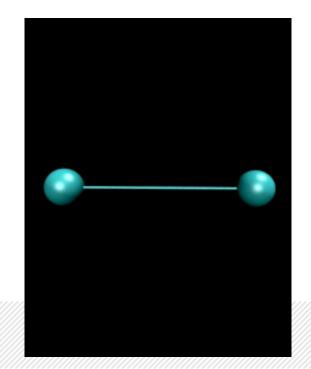


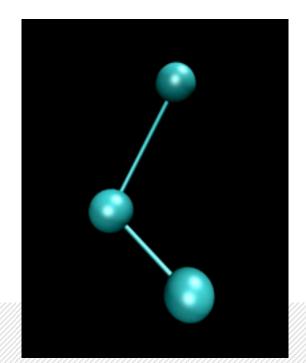
Common Potentials in Molecular Models

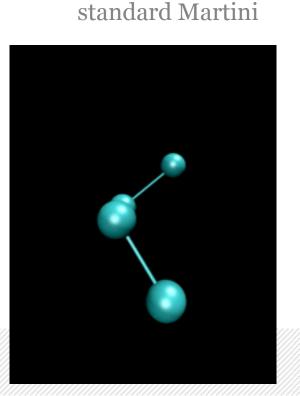
> BONDED: Simple harmonic/cosine bonds & angles

*
$$V(d) = \frac{k}{2} (d - d_0)^2$$
 $V(\theta) = \frac{k_{\theta}}{2} (\theta - \theta_0)^2$ * $V(\phi) = \sum_{n} K_n \left[1 + \cos(n\phi + \delta) \right]$

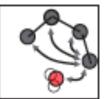
$$V(d) = \frac{k}{4} (d^2 - d_0^2)^2$$
 * $V(\theta) = \frac{k_{\theta}}{2} (\cos\theta - \cos\theta_0)^2$ * type used in standard Martini





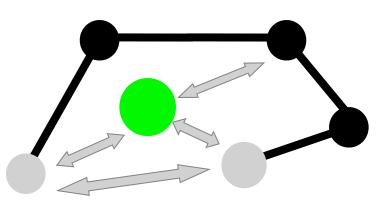




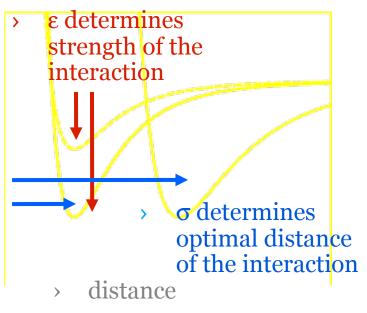


Common Potentials in Molecular Models

NON-BONDED: Coulomb interaction between partial charges (electrostatics)
 and Lennard-Jones potential (long-range dispersion and short-range repulsion)



$$E_{Coulomb} = \frac{q_1 q_2}{r_{12}} \qquad E_{LJ} = E_{dispersion} + E_{repulsion}$$
$$= 4\varepsilon \left(-\frac{\sigma^6}{r_{12}^6} + \frac{\sigma^{12}}{r_{12}^{12}}\right)$$



* Martini uses modified, smoothed variants of these!

- Usually simple functions with some physical justification
 - > Full descriptions of force fields implemented in GROMACS are described in the manual, Chapter 4

energy

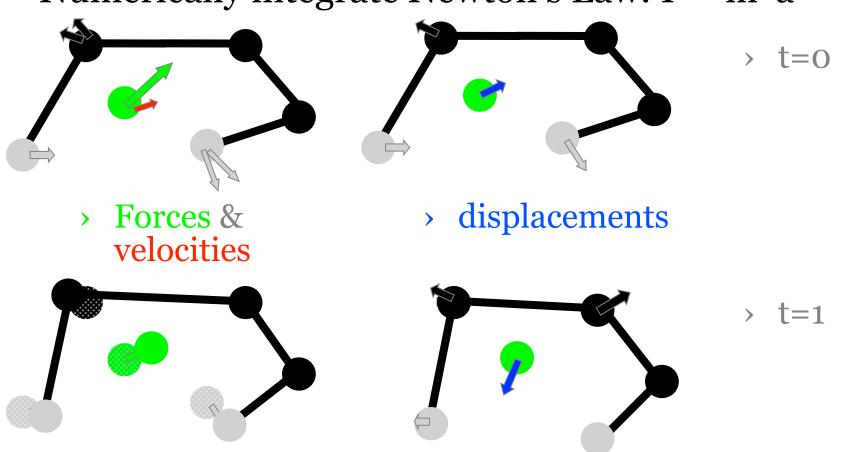


$$\Delta r = \dot{r}\Delta t + \frac{F}{2m}(\Delta t)^2$$

Basic MD Algorithm $ma = m\ddot{r} = F(r) = -\nabla E(r)$

$$ma = m\ddot{r} = F(r) = -\nabla E(r)$$

> Numerically integrate Newton's Law: F = m*a

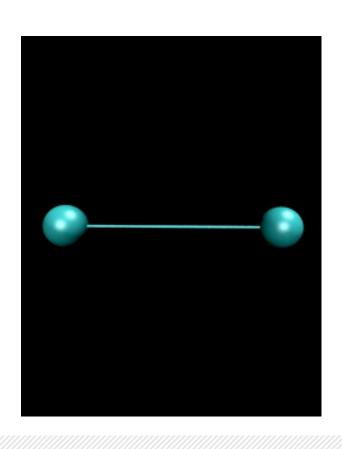


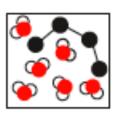
new displacements, etc...

> What goes in...

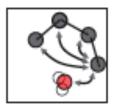
A Very Simple System

> Single oscillator treated classically in vacuo

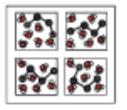




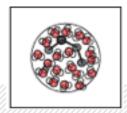




$$V(\mathbf{R}) = \frac{1}{2} k (\mathbf{R} - \mathbf{R}_0)^2$$



constant E



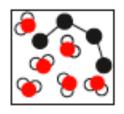
vacuum



> What goes in... using Gromacs

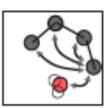
A Very Simple System





coordinates

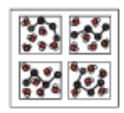
.gro or .pdb



$$V(\mathbf{R}) = \frac{1}{2} k (\mathbf{R} - \mathbf{R}_0)^2$$

.top
or .itp

definition of the system and of the molecules and their bonded and nonbonded interactions

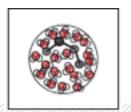


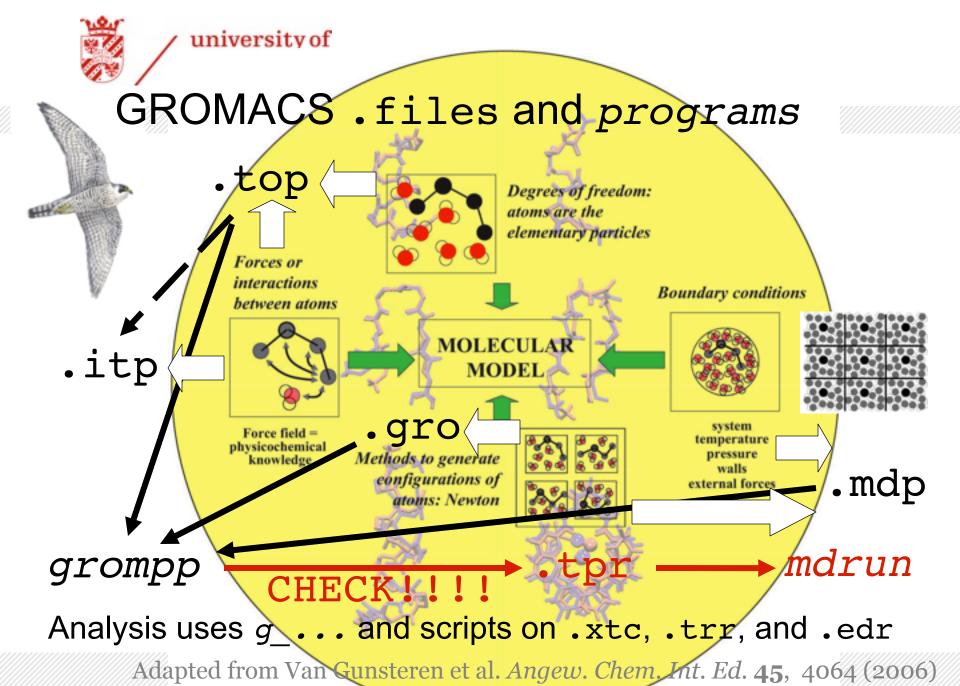
constant E

vacuum

.mdp

also algorithm for integration, number of steps, time step Δt , how often should we write output, and many other options!



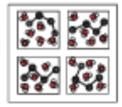




university of > What comes out... using Gromacs

Time-traces

- > Descriptions of the system
 - Positions
 - Velocities
 - Forces
 - Energies
 - Other system information
 - Other exetem informati
- > The user must then extract this information and visualize it
 - Gromacs has many tools for this!
 - Always VISUALLY inspect your simulation (ngmx, VMD, Pymo1, ...)



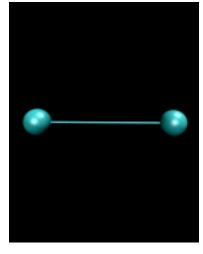
.xtc

.trr

.edr

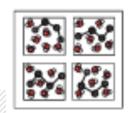
and/or





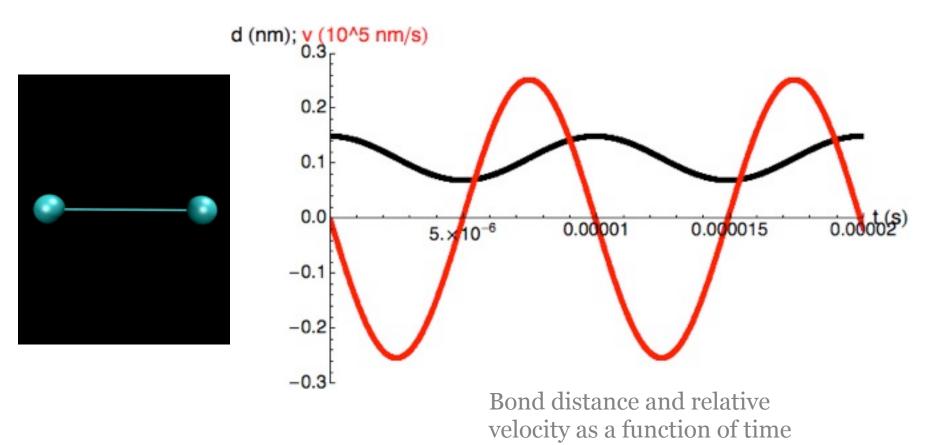


g_bond



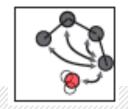
Visualization of some results

> Time-trace data can extracted and visualized





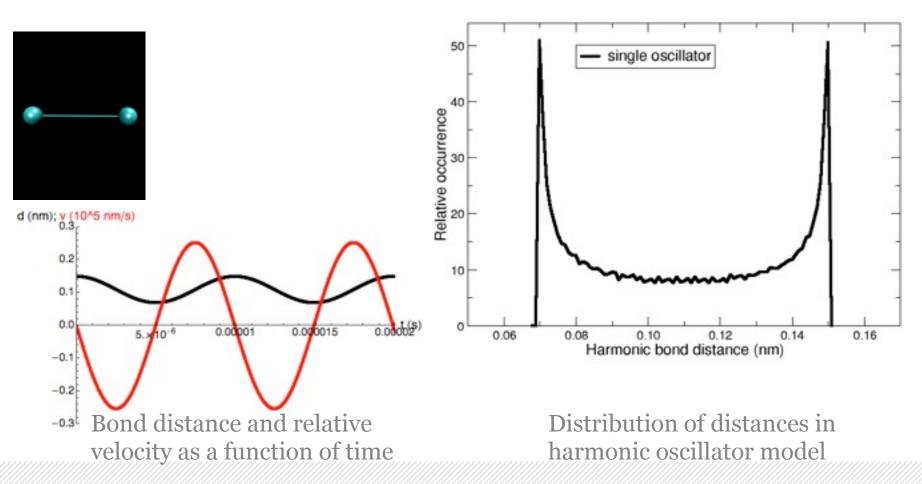
g_bond





Sampling a Distribution

> Time-trace data can be collected in a distribution





Useful and Informative Description(s)

> Relevant or interesting degrees of freedom

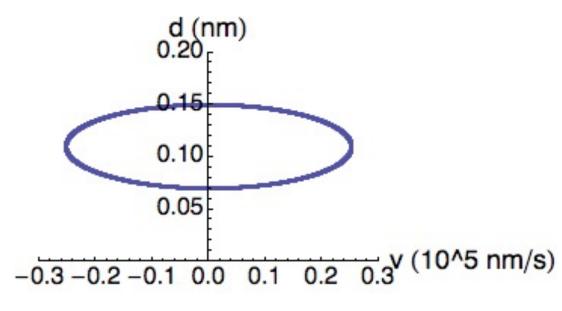
> Harmonic oscillator traces out a simple path

described by

Distance

Relative velocity

For an oscillator at constant energy, the ellipse constitutes its phase space. If the energy is not constant, many ellipses span its phase space, characterized by different amplitudes. (The path is traversed in counter clock-wise fashion.)

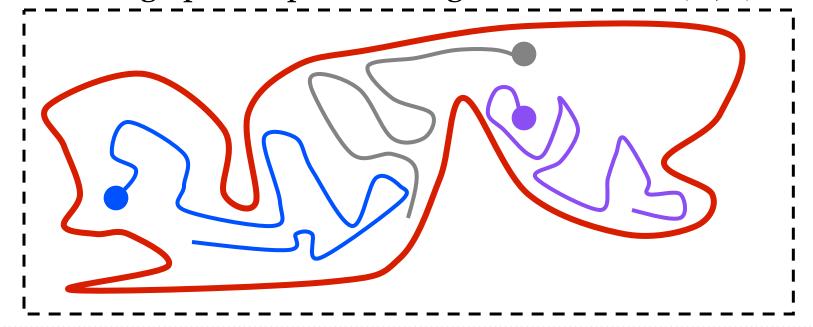


Distance (d) versus relative velocity (v) for a simple harmonic oscillator.

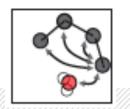


A Tasting (Sampling) of Classical Phase Space

- > Describes possible states of the system
 - Dimension $2*N_{dof}(\mathbf{R},\mathbf{P})$ or $N_{dof}(\mathbf{R})$
 - A single system, in time, follows an allowed path through phase space visiting allowed states (**R**,**P**)



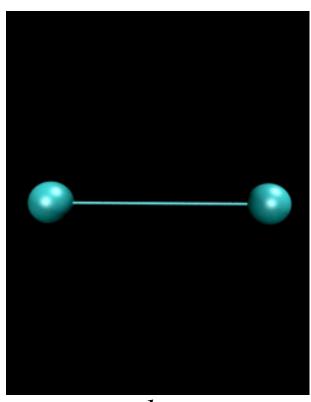




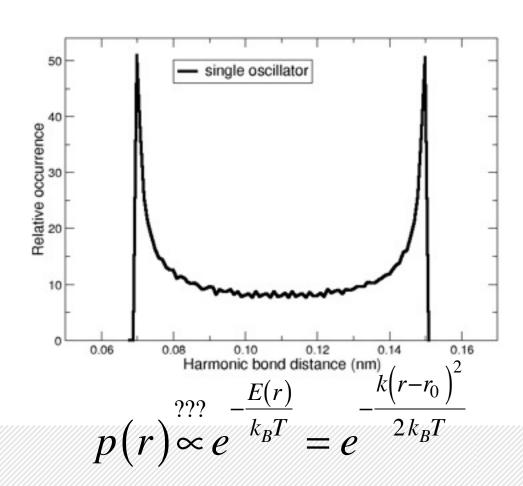


Potential and Distribution

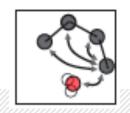
> The Boltzmann distribution is **not** observed for a single harmonic oscillator in vacuo



$$E(r) = \frac{k}{2} (r - r_0)^2$$



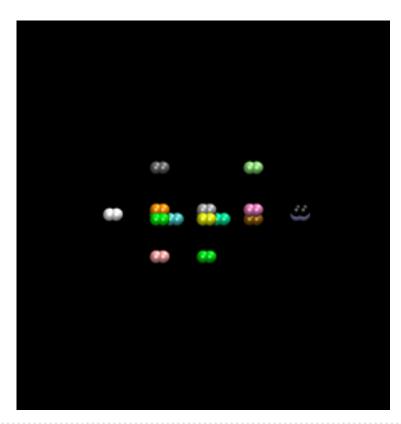


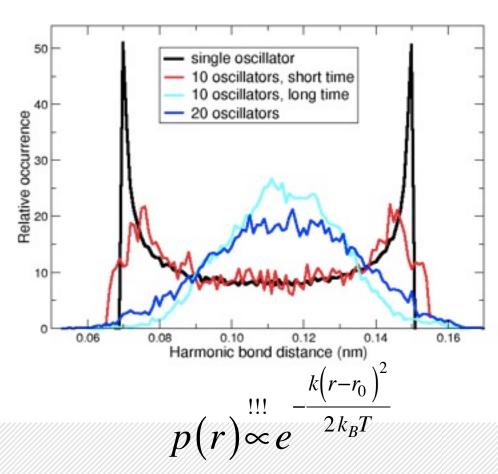




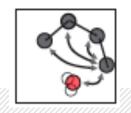
Potential and Distribution

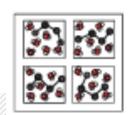
> The Boltzmann distribution is observed for anharmonically coupled oscillators





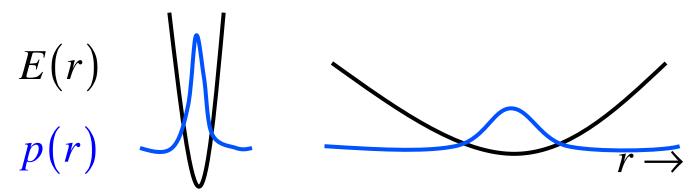






Potential and Distribution

Close link between potential and distribution



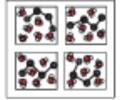
Can be used to extract potential from structural data

$$p(r) = \frac{e^{-\frac{E(r)}{k_B T}}}{Q} \iff E(r) = -k_B T \ln[p(r)Q]$$

- Inverse Boltzmann techniques
- In MARTINI for fine-tuning bonded interactions

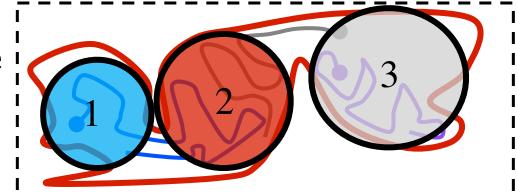
$$Q = \sum e^{-\frac{E_i}{k_B T}} \to Z = \int e^{-\frac{E(\mathbf{r})}{k_B T}} d\mathbf{r}$$





What we want to sample phase space for

- Define states as regions in phase space
 - Several realizations in the same state
 - Free energy difference between two states is related to the ratio of the number of realizations in each state and/or the probabilities of finding the system in a conformation belonging to a state

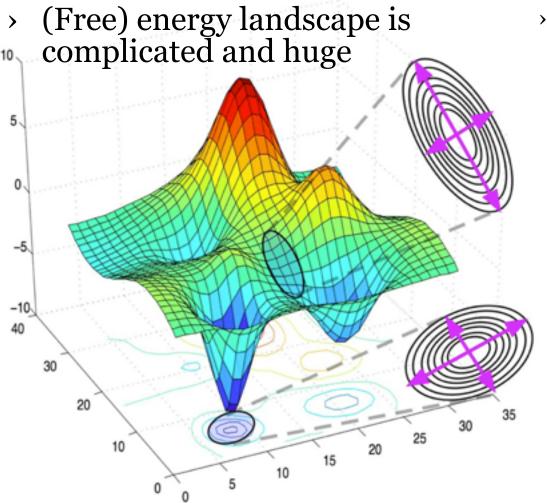


$$K_{12} = \frac{p_2^{eq}}{p_1^{eq}} = \frac{\sum_{i \in 2} e^{-\beta E_i}}{\sum_{j \in 1} e^{-\beta E_j}} = e^{-\beta \Delta G_{12}^0}$$

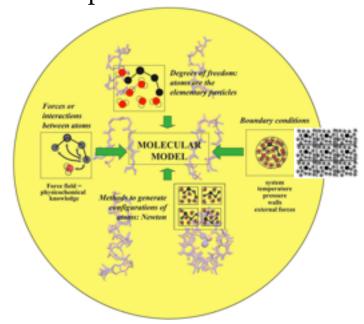
> The folded state of a protein is not a single conformation

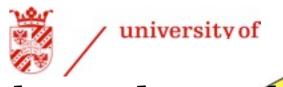


Sampling: a Problem?!



Obtaining a complete picture of the possible conformations and their probability of occurring is hampered by the sheer number of them and the barriers between local minima in the (free) energy landscape



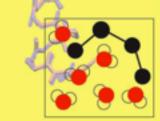


Enhanced Sampling Opportunities

Coarse-graining:

reducing the number of degrees of freedom, preserving the relevant

physics



Degrees of freedom: atoms are the elementary particles

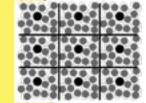
MOLECULAR MODEL

Multiscaling:

reducing detail in the surroundings leading to effective interactions

Boundary conditions





system temperature pressure walls external forces

Jumping:

exchanging snapshots between conditions to overcome barriers

between atoms

Force field = physicochemical knowledge

Methods to generate configurations of atoms: Newton

























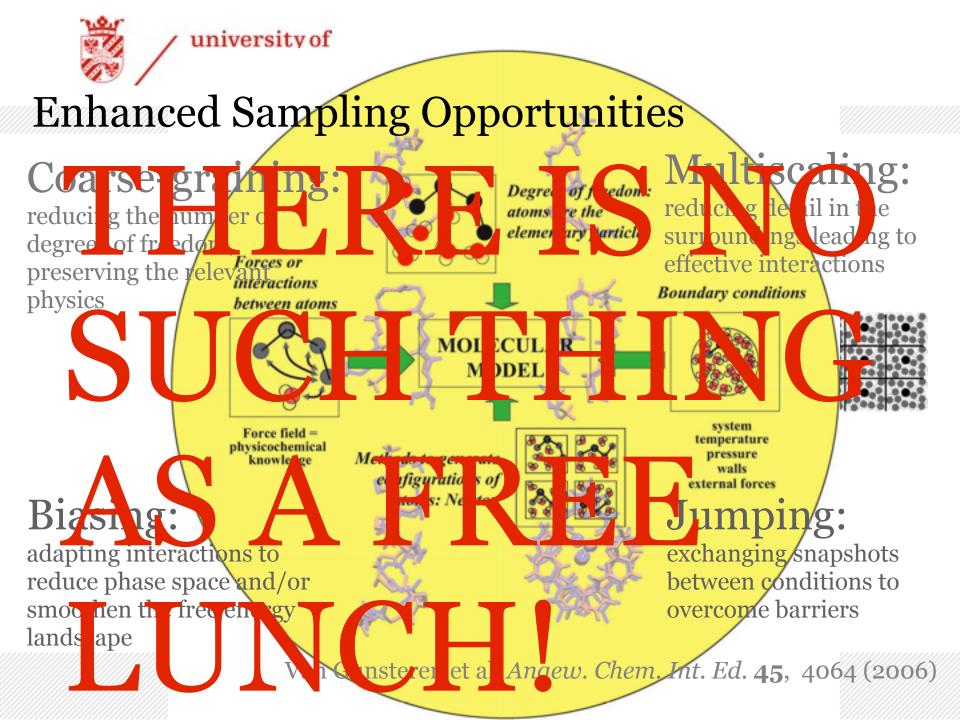
adapting interactions to reduce phase space and/or



smoothen the free energy landscape

Biasing:

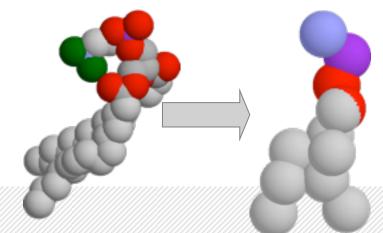
Van Gunsteren et al. *Angew. Chem. Int. Ed.* **45**, 4064 (2006)





COARSE GRAINING BENEFITS

- > Reduced Complexity
 - Physics: not all detail is relevant for our question
- > Efficiency: increase length and time scales
 - Space: reduced density reduces number of interactions, e.g. in 4-to-1 mapping
 - 4 (number) x 4 (pairs)
 - 4 neighbor searching
 - Time: smoother energy landscape, increased time step
 - in the algorithm: 10-20
 - effective time: 4
 - Total: 2.5-5 10³ speed-up





Martini Workshop 2015

COARSE GRAINING HowTo

Alex de Vries

A Simple(?) Example

Coarse-graining Caveats

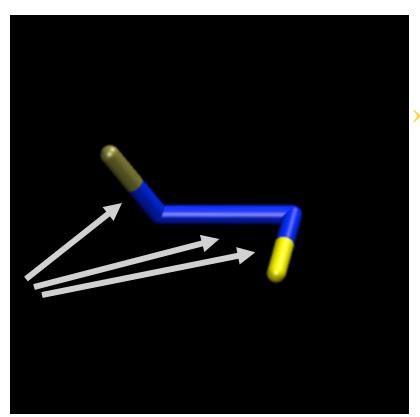
Coarse-graining Philosophies



- > 2-to-1 mapping scheme unto simple oscillator
 - Weak coupling through collisions (gas)

Weak LJ potential

Harmonic springs

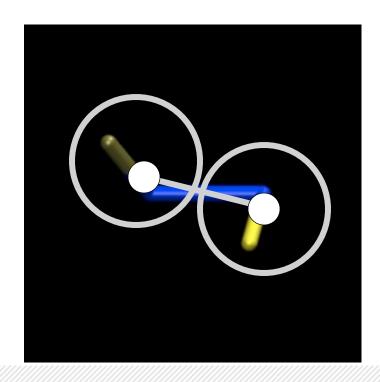


Small mass

Large mass



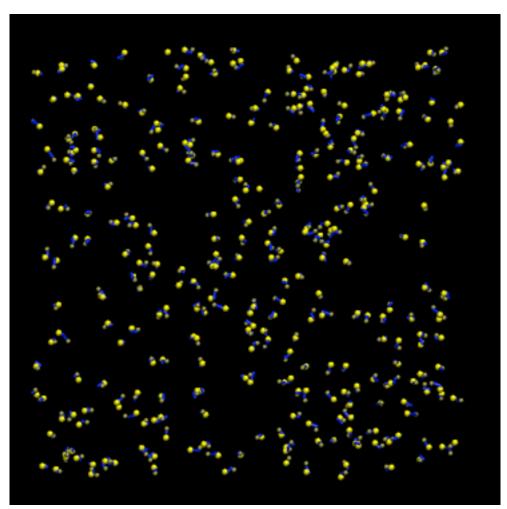
- > 2-to-1 mapping scheme unto simple oscillator
 - Coarse grain on two centers of mass





> THE REFERENCE

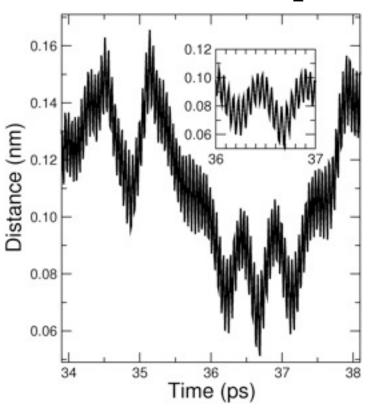
- A look at 100 ps trajectory for the system (looped)
- Energy exchange through collisions
- Run at constant Energy



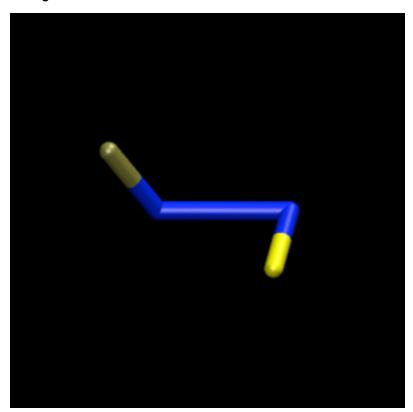


> COARSE-GRAINED BEHAVIOR

• A look at 10 ps trajectory for one oscillator

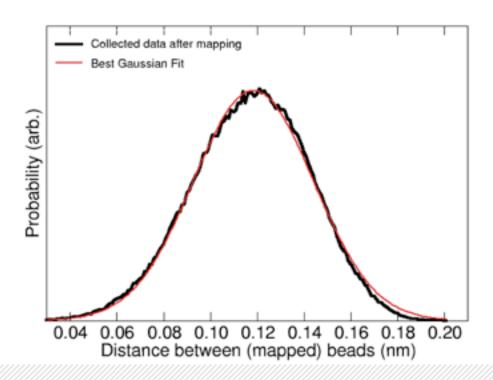


Distance between CG centers after 2-to-1 mapping





- > COLLECT DISTRIBUTIONS OF INTEREST
 - Distribution reflects effective interaction
 - Can possibly be achieved by a simple potential





$$E(r) = \frac{k}{2} (r - r_0)^2$$

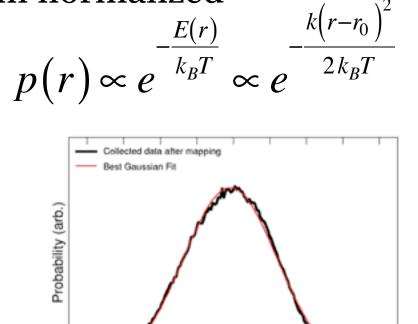
DERIVING POTENTIAL

> Harmonic force constant from normalized Gaussian distribution E(r)

$$\int \frac{1}{\sigma \sqrt{\pi}} e^{-\frac{\left(r-r_0\right)^2}{\sigma^2}} dr = 1$$

> Fit for best r_o and σ gives:

$$k = \frac{2k_BT}{\sigma^2}$$



$$p(r) = \frac{e^{-\frac{E(r)}{k_B T}}}{Q} \iff E(r) = -k_B T \ln[p(r)Q]$$

0.06 0.08 0.10 0.12 0.14 0.16 0.18 0.20 Distance between (mapped) beads (nm)

$$E(r) = \frac{k}{2} \left(r - r_0 \right)^2$$

DERIVING POTENTIAL

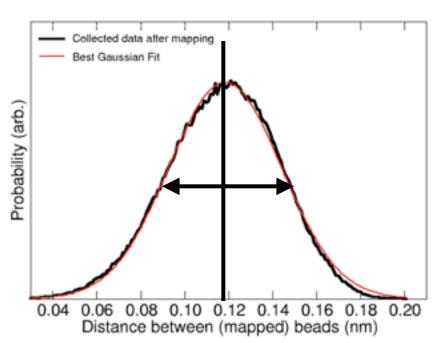
> Harmonic force constant from Gaussian distribution $k(r-r_0)^2$

$$p(r) \propto e^{-\frac{(3)}{2k_BT}}$$

> Fit for best r_0 and σ :

$$k = \frac{2k_BT}{\sigma^2}$$

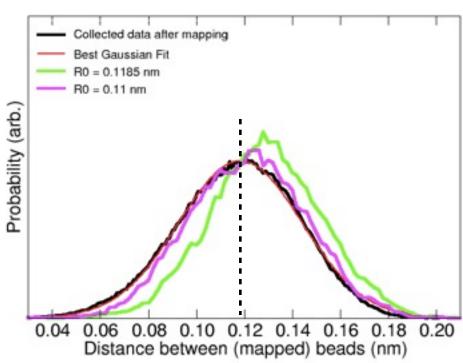
 $k = 1,269 \text{kJ} \cdot \text{mol}^{-1} \cdot \text{nm}^{-2}$ $r_0 = 0.1185 \text{nm}$







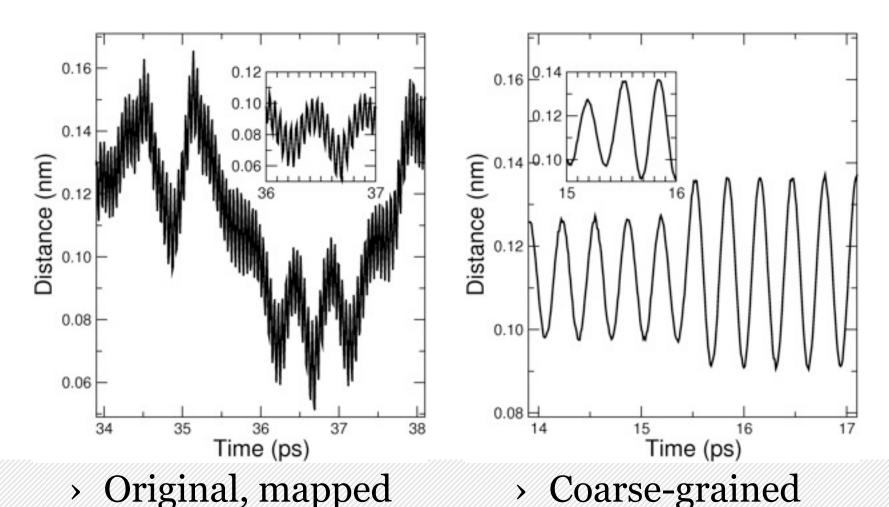
- Compare distributions at CG level
 - In this simple(?) example, there are already some complications
- Frequent collisions required to get good statistics
- Need to account for rotation that increases the bond length because of centrifugal force (*correlations* between degrees of freedom)
- Must use $r_o \approx 0.11$ nm in CG model instead of the 0.1185 nm found by fitting
- K and r_o will depend on total energy (temperature)



W.G. Noid et al, e.g. C.R. Ellis et al. *Macromol. Theory Simul.* **20**, 478 (2011)



Smoother interaction, smoother motion







> THE MEANING OF TIME

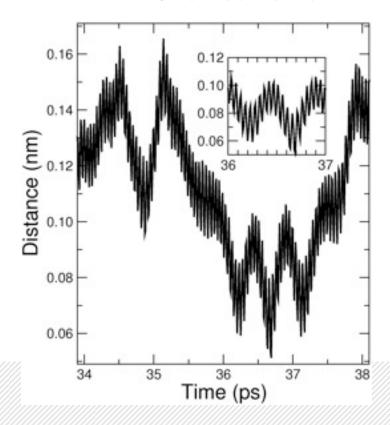
- Smoother interaction, smoother motion
 - Enables larger time steps
 - Friction is lower, sampling speeds up
 - Barriers are more easily overcome

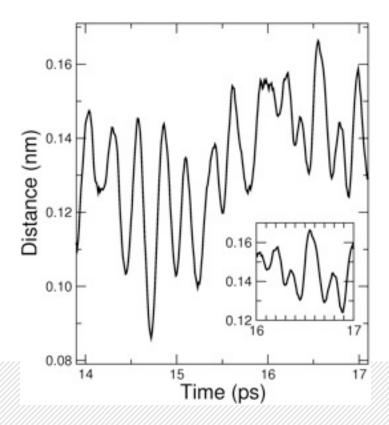
> DETAIL IS LOST

- Physics may be different
- Need to be careful in interpreting dynamics
- Exchange of energy between modes may be less efficient

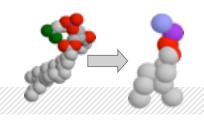
$$\Delta r = \dot{r}\Delta t + \frac{F}{2m}(\Delta t)^2 + \xi$$

- > THE MEANING OF TIME
 - Re-introduce friction through stochastic term
 - Part of benefit is gone, but dynamics may appear more realistic









Coarse-graining Philosophies

- > HIERARCHICAL MODELING
 - From quantum mechanics to evolution of galaxies
 - Interactions at less detailed level are the result of the collective interactions at more detailed level
 - General method applicable to any system (like an algorithm)
- > EFFICIENT MODEL AT CERTAIN SCALE
 - Reproduce faithfully certain chosen properties
 - Developed with certain application in mind
 - Nevertheless aiming at wide use through considering the physics of the problems in mind



Hierarchical modeling: pros and cons

- > PRO
 - UNBIASED
 - Physics follows through the hierarchy of models
 - STRAIGHTFORWARD MULTISCALING
 - Enables reliable combination of levels of modeling
 - Entirely general approach

> CON

- REQUIRES LARGE WORK LOAD
 - Need detailed level simulations to derive CG potentials
 - Complicated numerical potentials
- LIMITED VALIDITY
 - Strictly valid for one state point only (new system, new potentials)



Semi-empirical modeling: pros and cons

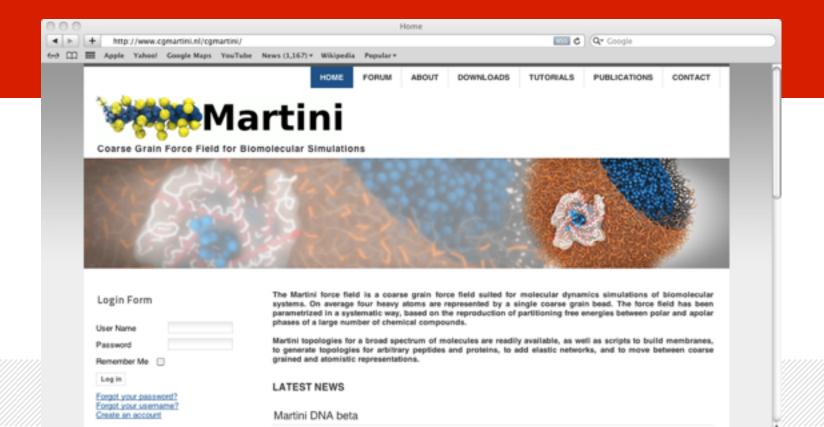
- > PRO
 - CHEAP
 - Parameterize on empirical data available
 - Simple analytical potentials
 - TRANSFERABLE
 - After parameterizing building blocks, many similar systems can be treated straightforwardly

> CON

- BIASED
 - Toward parameterized properties
- PROBLEMATIC MULTISCALING
 - Different levels do not need to correspond closely
 - Extensive validation required



MARTINI MODEL IS A SEMI-EMPIRICAL FORCE FIELD





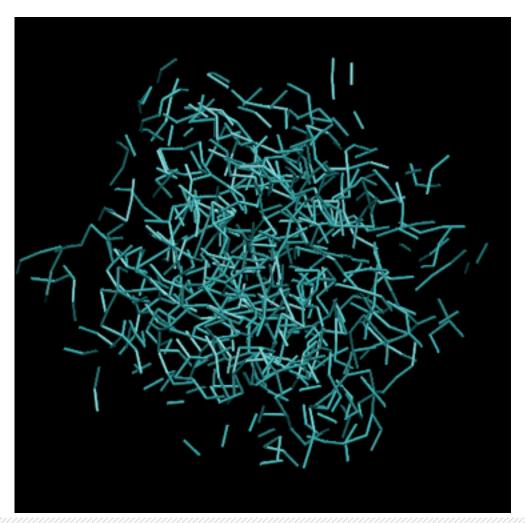
What do we expect of the Martini CG model?

- For an empirical CG model
 - Results of complex systems should be consistent with experiment even though not explicitly parameterized on that complex system
- > MARTINI does pretty well for lipids
 - Many examples are described in literature
- > MARTINI extensions
 - Proteins
 - Sugars
 - Polymers



Example: liquid hexadecane

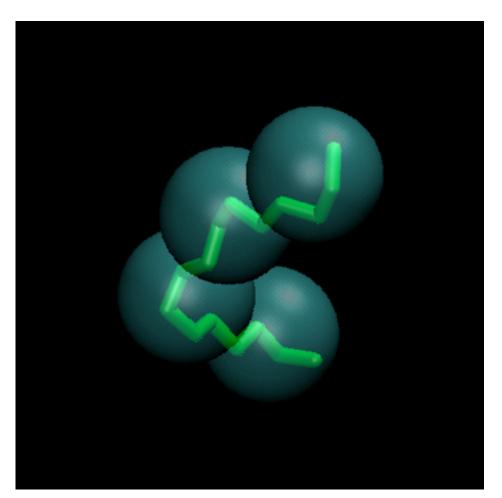
- Standard Martini model* compared to mapped GROMOS 53A6 model
- Movie shows CG representation and is looping over 0.2 ns
- 320 hexadecane molecules in the system
- T = 300 K





Example: liquid hexadecane

- Movie is looping over 5 ns
- 1 hexadecane molecule of 320 in the system
- Overall translation and rotation of the molecule is removed

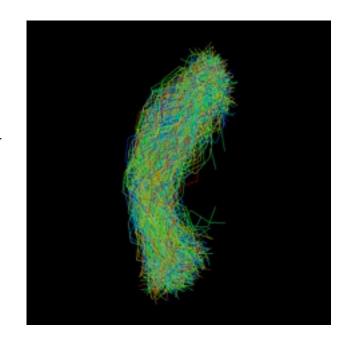




Characterizing a collection of structures

- > Are conformations realistic?
- Time scale of sampling
- Schlitter's formula for configurational entropy
 - Upper bound
 - Approximation for harmonic oscillator

$$S_{true} \le S = \frac{k_B}{2} \ln \left\| 1 + \frac{k_B T e^2}{\hbar^2} \right\|$$



- > Procedure
 - Fit (part of) the structure to remove translation (and rotation)

Mass-weighted covariance matrix

| Juniversity of groningen |
$$S_{true} \le S = \frac{k_B}{2} \ln \left| 1 + \frac{k_B T e^2}{\hbar^2} \right|$$
 | Mass-weighted covariance matrix | $1 + \frac{k_B T e^2}{\hbar^2}$

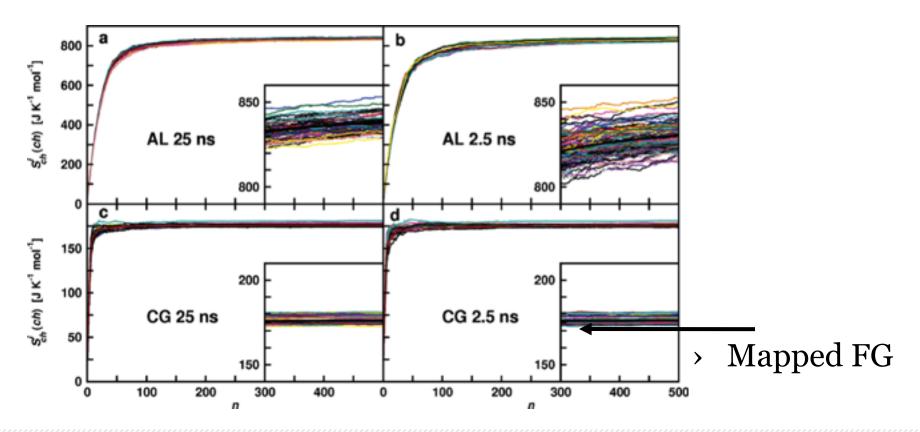
$$\mathbb{D} = \begin{pmatrix} \frac{m_1}{K} \sum_{k} (r_1^k - \langle r_1 \rangle)^2 & \dots & \frac{1}{K} \sum_{k} (r_1^k - \langle r_1 \rangle) (r_N^k - \langle r_N \rangle) \\ \vdots & \ddots & \vdots \\ \frac{1}{K} \sum_{k} (r_N^k - \langle r_N \rangle) (r_1^k - \langle r_1 \rangle) & \dots & \frac{m_N}{K} \sum_{k} (r_N^k - \langle r_N \rangle)^2 \end{pmatrix}$$

- > Here, *K* is the number of conformations
- $\rightarrow r_1^k$ is the position of atom/bead number 1 in frame k
- $\rightarrow m_1$ is the mass of atom/bead 1
- Note similarity to variance!

$$S_{true} \le S = \frac{k_B}{2} \ln \left| 1 + \frac{k_B T e^2}{\hbar^2} \mathbb{D} \right|$$

Configurational Entropy

- MARTINI versus GROMOS 53A6
 - Studied at FG (AL) and CG levels by Baron et al.



Baron et al *J. Phys. Chem. B* **110**, 8464 (2006)

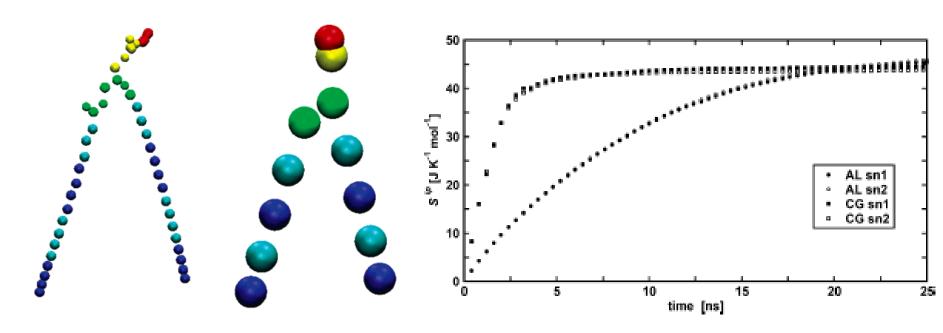
$$S_{true} \le S = \frac{k_B}{2} \ln \left| 1 + \frac{k_B T e^2}{\hbar^2} \mathbb{D} \right|$$

Configurational Entropy: Time Scale

> For phospholipid DPPC in bilayer

Average structures

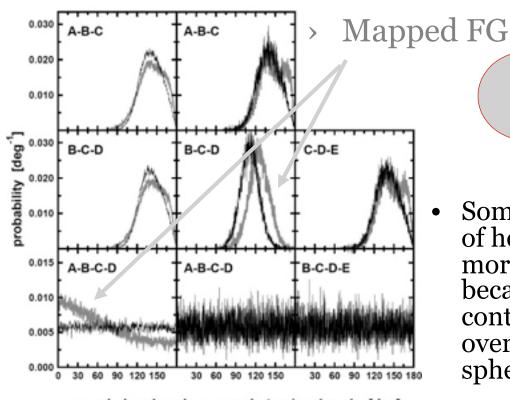
Build-up of sampling after mapping



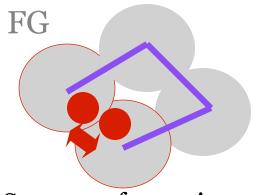


Bonded interactions: compare to FG model

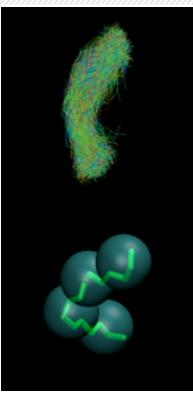
- MARTINI versus GROMOS 53A6
 - Bonded distributions MARTINI and mapped FG differ







Some conformations of hexadecane are more favorable in FG because of short contacts but overlapping vdW spheres in CG





COARSE GRAINING SUMMARY

- > SPEEDING UP SEARCH IN PHASE SPACE
 - Use an effective interaction from distribution
 - Smooths Potential Energy Surface
 - Reduces number of interactions
 - Increases effective time
- > DETAIL IS LOST
 - Beneficial: detail may not be required
 - Detrimental: some system characteristics are gone



Thank you for your attention

$$\mathbf{p}_{i}(t) = m_{i} \frac{d\mathbf{r}_{i}(t)}{dt}; \quad \mathbf{F}_{i}(t) = \frac{d\mathbf{p}_{i}(t)}{dt}$$

Integrating equations of motion

> To compute a new position after a time interval Δt

$$\mathbf{r}_{i}\left(t+\Delta t\right) = \mathbf{r}_{i}\left(t\right) + \int_{t}^{t+\Delta t} \frac{\mathbf{p}_{i}\left(\tau\right)}{m_{i}} d\tau \approx \mathbf{r}_{i}\left(t\right) + \frac{\overline{\mathbf{p}}_{i}\left(t\right)}{m_{i}} \Delta t$$

> Similarly, for the new momentum

$$\mathbf{p}_{i}\left(t+\Delta t\right) = \mathbf{p}_{i}\left(t\right) + \int_{t}^{t+\Delta t} \mathbf{F}_{i}\left(\tau\right) d\tau \approx \mathbf{p}_{i}\left(t\right) + \overline{\mathbf{F}}_{i}\left(t\right) \Delta t$$

 \rightarrow Combining these from t = 0

$$\mathbf{r}_{i} \left(\Delta t\right) = \mathbf{r}_{i} \left(0\right) + \frac{1}{m_{i}} \int_{0}^{\Delta t} \left[\mathbf{p}_{i} \left(0\right) + \int_{0}^{\tau} \mathbf{F}_{i} \left(\tau'\right) d\tau'\right] d\tau$$

$$\approx \mathbf{r}_{i} (0) + \frac{\overline{\mathbf{p}}_{i} (0)}{m_{i}} \Delta t + \frac{\overline{\mathbf{F}}_{i} (0)}{2m_{i}} (\Delta t)^{2}$$

Verlet Integrator

$$\mathbf{r}(t+\Delta t) = \mathbf{r}(t) + \frac{d\mathbf{r}}{dt}\Big|_{t} \Delta t + \frac{1}{2!} \frac{d^{2}\mathbf{r}}{dt^{2}}\Big|_{t} (\Delta t)^{2} + \dots$$

$$\mathbf{r}(t - \Delta t) = \mathbf{r}(t) - \frac{d\mathbf{r}}{dt}\Big|_{t} \Delta t + \frac{1}{2!} \frac{d^{2}\mathbf{r}}{dt^{2}}\Big|_{t} (\Delta t)^{2} - \dots$$

$$\mathbf{r}(t+\Delta t)+\mathbf{r}(t-\Delta t)=2\mathbf{r}(t)+\frac{d^2\mathbf{r}}{dt^2}\bigg|_{t}(\Delta t)^2+O((\Delta t)^4)$$

$$\Rightarrow \mathbf{r}(t + \Delta t) + \mathbf{r}(t - \Delta t) \approx 2\mathbf{r}(t) + \frac{\mathbf{F}}{m}(\Delta t)^{2}$$

Taylor expansion of the position in time, assuming constant force



$$\mathbf{F} = m \frac{d^2 \mathbf{r}}{dt^2}$$

Verlet integrator from Newton's Law

$$\left[\mathbf{r}(t+\Delta t)-\mathbf{r}(t)\right]-\left[\mathbf{r}(t)-\mathbf{r}(t-\Delta t)\right]=\frac{F(t)}{m}\Delta t^{2}$$

$$\mathbf{r}(t + \Delta t) - \mathbf{r}(t) \approx \frac{\mathbf{F}}{m}(\Delta t)^{2}$$

$$\mathbf{r}(t + \Delta t)$$

$$\mathbf{r}(t + \Delta t) - \mathbf{r}(t)$$

$$\mathbf{r}(t) - \mathbf{r}(t - \Delta t)$$

$$\mathbf{r}(t) - \mathbf{r}(t)$$

$$\mathbf{r}(t) - \mathbf{r}(t)$$

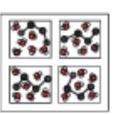
$$\mathbf{r}(t) - \mathbf{r}(t)$$

$$\mathbf{r}(t) - \mathbf{r}(t)$$

 $\frac{d^2\mathbf{r}}{dt^2}$ is the difference of the difference, or the change in the change

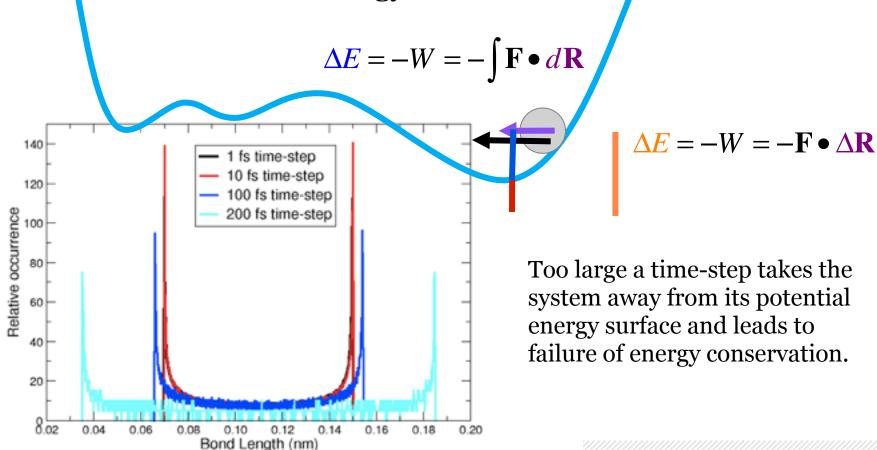






Conservation of Energy

In practice, even with small time-steps,
 conservation of energy is difficult to achieve





$$\Delta r = \dot{r}\Delta t + \frac{F}{2m}(\Delta t)^2$$

Practical considerations: Conservation of Energy

> In numerical approaches, time-step should be such that PES is followed

$$ma = m\ddot{r} = F(r) = -\nabla E(r)$$

$$\Delta E = -F(r) \cdot \Delta r$$

$$\Delta r \approx \frac{F(r)(\Delta t)^{2}}{2m}$$

We should end up at the potential energy we expect or energy is lost from/added to the system

$$\Delta E_{pred} \approx -F(r) \bullet \frac{F(r)(\Delta t)^2}{2m}$$

$$\dot{r} = v = \frac{dr}{dt}; \quad \ddot{r} = a = \frac{d^2r}{dt^2}$$

$$E(r)$$
 and $U(r)$ are used interchangeably

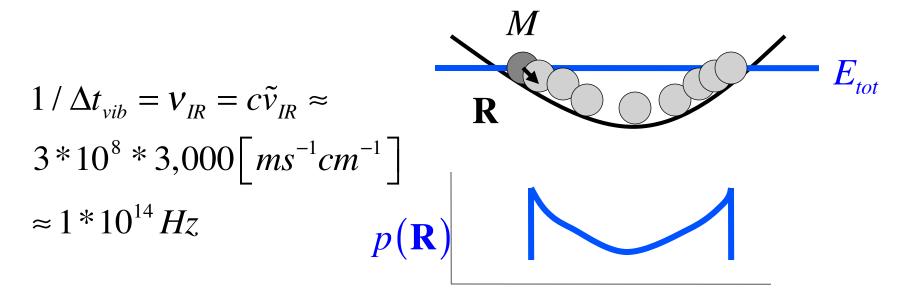






Appropriate time step for sampling

> At atomistic level, time step for fastest vibrations

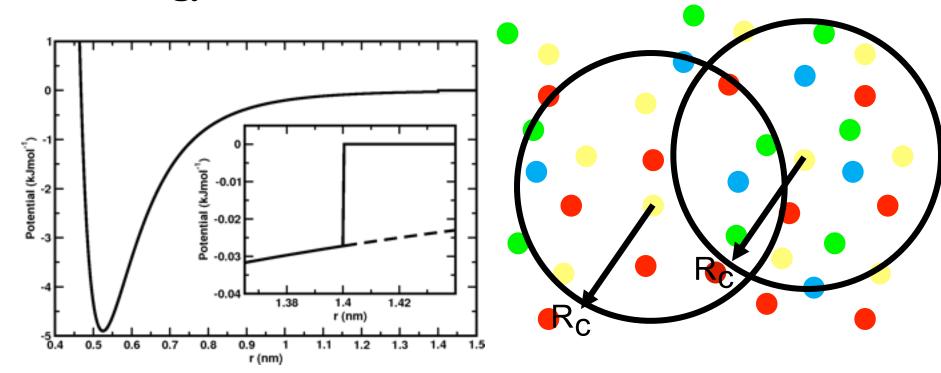


• Time step ~1 fs



Cut-off noise

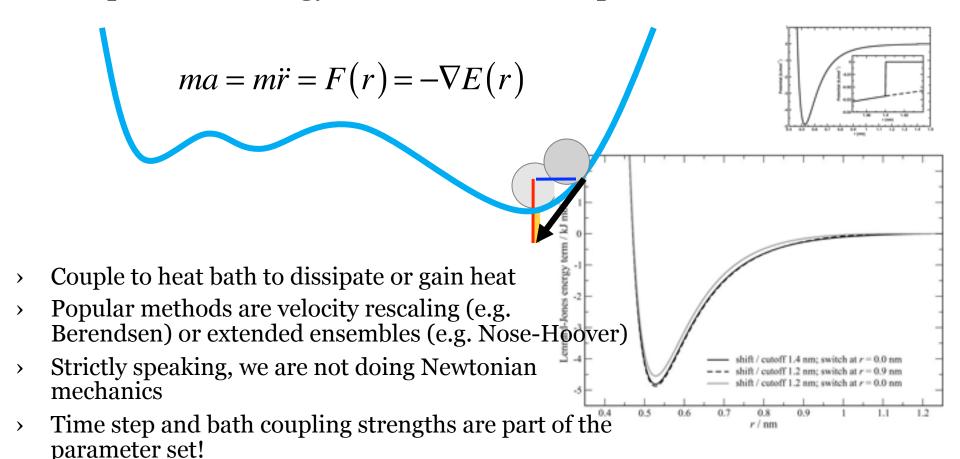
 Particles start/stop interacting at certain distance: energy is not conserved



$$\Delta r = \dot{r}\Delta t + \frac{F}{2m}(\Delta t)^2$$

Practical considerations: Conservation of Energy

> In practice, energy conservation is impossible to achieve



 Martini uses non-bonded potentials without jump at cut-off (shifted potentials)